Design of Linear Phase Selective Comb-Line Filter

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Abstract — **A multiwire approach has been used to develop design equations for linear phase selective comb-line filter. The filter under consideration consists of two rows of inductive resonators separated by a slotted coupling surface. Through the development process a multipath prototype network has been considered.**

A frequency transformation has been formulated to relate the multipath prototype network and the multiwire comb-line structure. Hence, once the element-values of the prototype network are chosen to meet prescribed requirements, the corresponding comb-line filter element values can be computed through the developed explicit formulas.

To illustrate the design procedure a brief design example is presented.

I. Introduction

THE ADVANCEMENT IN communication systems, especially in high capacity communication channels and digital transmission, imposes the need for filters with selective amplitude and linear phase characteristics. The approximation and the optimization problems which are related to the design of filters with high performance had been discussed by the author in [1]. The transfer function obtained by the use of the generalized Chebyshev rational function in [1] has been synthesized in the form of a multipath network. In this paper the prototype network of the above paper which is shown in Fig. 1 is to be designed in the comb-line form.

The theory of comb-line bandpass filters has been studied extensively in the technical literature, [2]. The available techniques deal with the realization of comb-line filters based upon prototype networks with minimum phase, i.e., the transfer functions that characterize these prototypes are minimum-phase functions. In this paper the lowpass prototype network is described by a nonminimum-phase function as illustrated in [3]. This prototype (shown in Fig. 1) is to be used here due to the following.

1) The amplitude and the phase responses of the prototype are compatible. The two capacitors at the input and the output ports produce two transmission zeros at infinity and this will provide a reasonable selective amplitude. The multipath nature of the network can also provide flat group delay (linear phase) over most of the entire passband.

2) The element-values of this prototype network are available for degrees up to and including 14 with different number of transmission zeros at infinity (minimum two). Tables of element values are provided in [1] and [4].

The prototype network of Fig. (1) has been realized in

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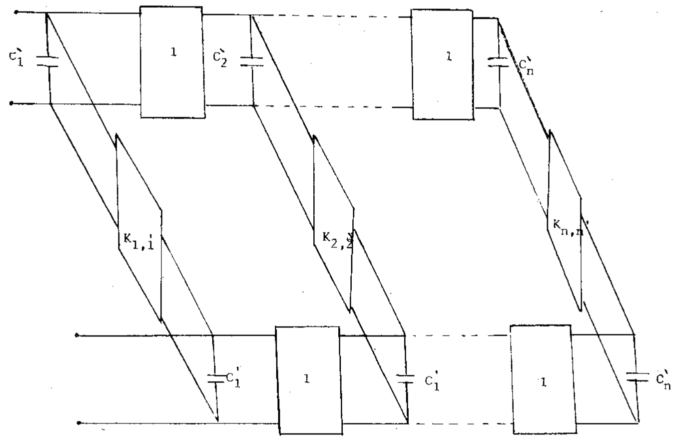


Fig. 1. Low-pass prototype network.

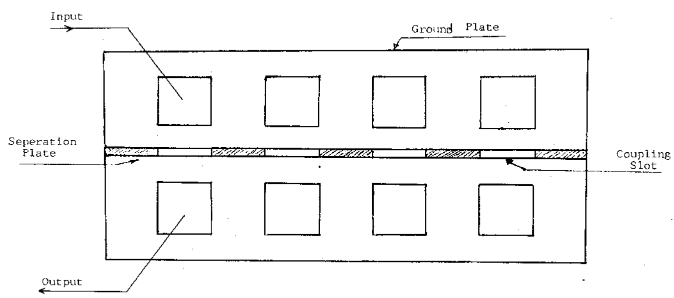


Fig. 2. Generalized comb-line linear phase filter with 8 resonators.

different forms, direct-coupled cavity [5], interdigital [6], and in-line dual-mode cavities [7]. In this paper the design equations for the generalized comb-line filter are to be presented. The generalized comb-line structure under consideration is shown in Fig. (2). The structure consists of two rows of coupled resonators, each row coupled to the other through slots in the coupling-plate that separates them. Each row composed of n-resonators and is similar to the minimum phase comb-line structure. That is, all the inductive resonators are shortened to the groundplate at one end while at the other the resonators are left open. Lumped capacitors as shown in Fig. (3) are used to provide resonance with the associated inductive resonators at the center of the passband to perform the filtering process.

For a prototype network of degree $(2n)$ and with $(2m)$ transmission zeros at infinity, the first $(m-1)$ crosscouplings do not exist. The comb-line that realizes the above prototype consists of n-resonators in each row with $(n+1$

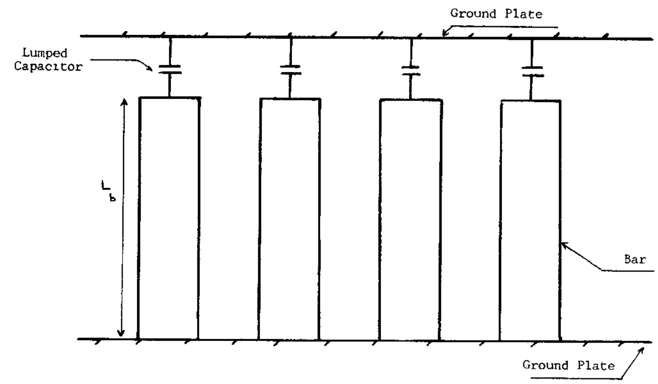


Fig. 3.

$-m$) slots in the separation plate.

In this paper the generalized theory of multiwire networks [8] has been used to develop the design equations of comb-line filter. Explicit formulas for the characteristic admittances of the lines that form the structure are developed in terms of the scaling factor α and the element values of the prototype network.

II. The Multiwire Networks And The Generalized Comb-Line Structure

The nodal analysis of multiwire network shows that the admittance matrix of the structure which consists of n lines can be defined by

$$\begin{bmatrix} [I_t] \\ [I_t'] \end{bmatrix} = \frac{1}{t} \begin{bmatrix} [Y] & -\sqrt{1-t^2}\,[Y] \\ -\sqrt{1-t^2}\,[Y] & [Y] \end{bmatrix} \begin{bmatrix} [V_t] \\ [V_t'] \end{bmatrix} \tag{1}$$

where

I_t, I_t', V_t, and V_t'	shown in Fig. 4;
t	$= \tanh(\beta p)$;
p	the complex frequency variable;
β	a constant; and
$[Y]$	$(n \times n)$ symmetrical matrix whose entries are determined from the static capacitances of and between the lines.

The comb-line under consideration is composed of $2n$ lines and $2n$-shunt lumped capacitors. The admittance matrix of the generalized comb-line filter can be presented by using (1) in the form shown (2)

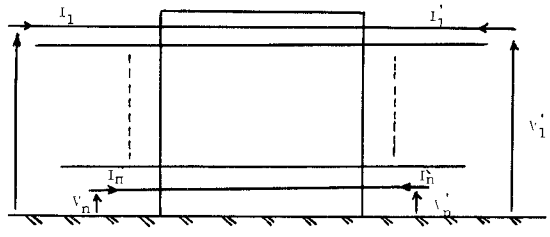
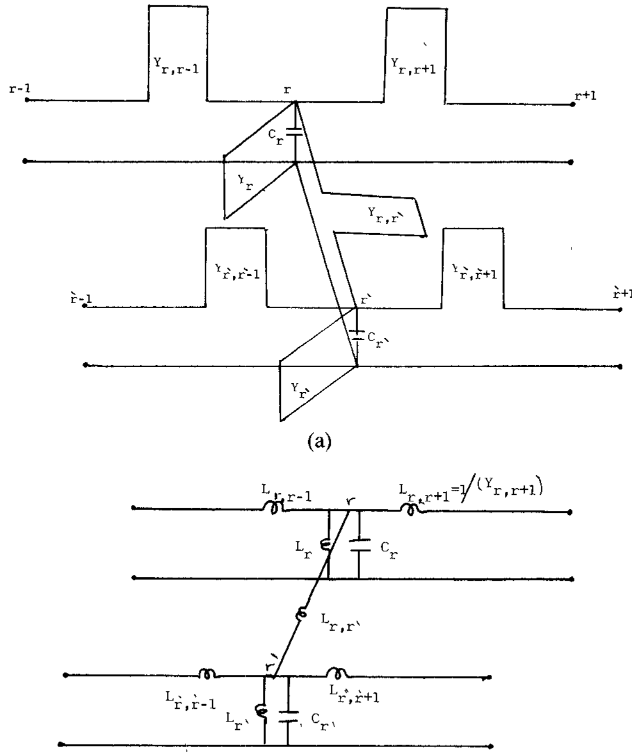


Fig. 4. Multiwire network.



(a)

(b)

Fig. 5.

where, referring to Fig. 5(a)

$$Y_{r,r} = Y_r + Y_{r-1,r} + Y_{r,r+1} + Y_{r,2n+1-r} \tag{3}$$

is the sum of all distributed capacitances (reciprocal inductances) emanating from node r, and C_r is the lumped capacity from node r, to ground and all the nonadjacent admittances are assumed to be zeros.

From (2) and (3), in conjunction with Fig. 5(a), the coupling between adjacent resonators (in the same row or

$$\begin{bmatrix} \frac{Y_{1,1}}{t}+PC_1 & \frac{-Y_{1,2}}{t} & 0 & \cdots & 0 & \frac{-Y_{1,2n}}{t} \\ \frac{-Y_{2,1}}{t} & \frac{Y_{2,2}}{t}+PC_2 & 0 & & \frac{-Y_{2,2n-1}}{t} & 0 \\ 0 & \frac{-Y_{3,2}}{t} & \frac{Y_{3,3}}{t}+PC_3 & \iddots & & \vdots \\ \vdots & & & \ddots & & 0 \\ 0 & \frac{-Y_{2n-1,2}}{t} & \iddots & & \frac{Y_{2n-1,2n-1}}{t}+PC_{2n-1} & \frac{-Y_{2n-1,2n}}{t} \\ \frac{-Y_{2n,1}}{t} & 0 & \cdots & 0 & \frac{-Y_{2n,2n-1}}{t} & \frac{Y_{2n,2n}}{t}+PC_{2n} \end{bmatrix} \tag{2}$$

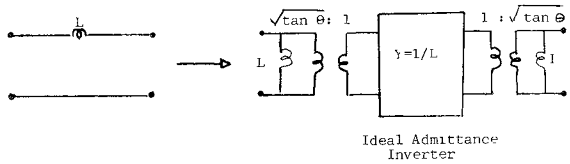


Fig. 6.

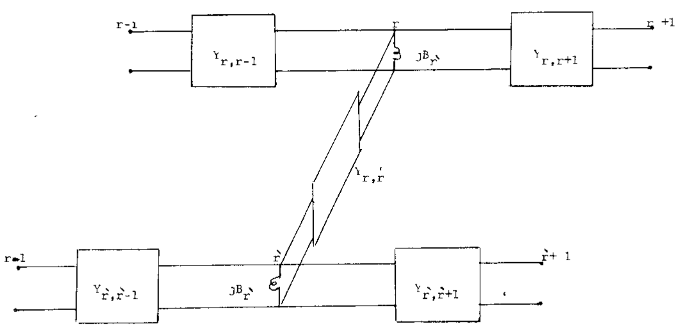


Fig. 7.

in the opposing one) is observed to be a series short-circuited stub, or distributed inductor. Based on the above, a typical section of the comb-line filter can be configurated as depicted in Fig. 5(a), (b).

By utilizing the technique presented in [9] a series short-circuited stub can be decomposed into an ideal immittance inverter, two transformers, and two shunt short-circuited stubs are shown in Fig. 6.

In Fig. 6, θ is the electrical length that corresponds to the real frequency $\omega = P/j$ and

$$t = j\tan\theta. \tag{4}$$

The typical section of the generalized comb-line filter after removing the transformers, reduces to the section shown in Fig. 7 where

$$B_r = (\omega C_r - Y_{r,r}\cot\theta)\tan\theta. \tag{5}$$

C_r is to be chosen to produce perfect transmission at the center of the passband, i.e., at $\theta = 0$, hence at

$$\theta = \theta_0 \qquad B_r = 0 \tag{6}$$

$$\omega_0 C_r - Y_{r,r}\cot\theta_0 = 0 \tag{7}$$

$$C_r = \frac{Y_{r,r}\cot\theta_0}{\omega_0}. \tag{8}$$

III. Frequency Transformation and Admittance Scaling

The typical shunt susceptance of the generalized comb-line filter is given by

$$B_r = (\omega C_r - Y_{r,r}\cot\theta)\tan\theta \tag{5}$$

and by substituting (8) in (5) B_r simplifies to

$$B_r = Y_{r,r}\left\{\frac{\omega}{\omega_0}\frac{\tan\theta}{\tan\theta_0} - 1\right\} \tag{9}$$

but

$$\frac{\omega}{\omega_0} = \frac{\theta}{\theta_0} \tag{10}$$

hence

$$B_r = Y_{r,r}\left\{\frac{\theta}{\theta_0}\frac{\tan\theta}{\tan\theta_0} - 1\right\}. \tag{11}$$

Therefore, the required transformation is given by

$$\omega' = \left(\frac{\theta}{\theta_0}\frac{\tan\theta}{\tan\theta_0} - 1\right)\alpha \tag{12}$$

where θ_0 are to be determined from the edges of the passband of the bandpass response, and ω' is the low-pass prototype frequency.

If $-1, 1$ are the edges of the low-pass prototype response in ω', and ω_1, ω_2 are the edges of the required bandpass response, then from (12)

$$-1 = \left\{\frac{\theta_1}{\theta_0}\frac{\tan\theta_1}{\tan\theta_0} - 1\right\}\alpha \tag{13}$$

$$1 = \left\{\frac{\theta_2}{\theta_0}\frac{\tan\theta_2}{\tan\theta_0} - 1\right\}\alpha \tag{14}$$

where θ_1 and θ_2 are the electrical lengths corresponding to ω_1 and ω_2, respectively.

Using (13) and (14)

$$\alpha = \frac{\theta_2\tan\theta_2 + \theta_1\tan\theta_1}{\theta_2\tan\theta_2 - \theta_1\tan\theta_1} \tag{15}$$

$$\theta_0\tan\theta_0 = \tfrac{1}{2}(\theta_2\tan\theta_2 + \theta_1\tan\theta_1) \tag{16}$$

and for narrow-band applications (16) can be simplified to

$$\theta_0 = \frac{\theta_1 + \theta_2}{2} \tag{17}$$

otherwise (16) must be solved numerically.

The inductive resonators of the comb-line structure are often chosen to be about 90° at a specified frequency in the stopband "f_c". Then the electrical lengths θ_1 and θ_2 can be calculated. However, θ_0 is determined by (16) and consequently the center of the passband f_0 can be computed.

By scrutinizing (15) it can be easily concluded that α is a large number for narrow bandwidth. However, for a good design, reasonable admittance values are necessary. An admittance level of unity for $Y_{r,r}$ can be achieved by scaling the admittance matrix of the $2n$-wire structure or the prototype network. The other reason for scaling $Y_{r,r}$ to unity is all the lumped capacitors will have the same value

$$C_r = \frac{\cot\theta_0}{\omega_0}. \tag{18}$$

A typical section of the low-pass prototype network is shown in Fig. 8 where all the coupling elements are ideal admittance inverters. By scaling the rth node by $1/(\alpha C_r)$, then all of the frequency dependent elements are assigned to the value $(1/\alpha)$. Hence the scaled values of the prototype network become

$$C_r'' = 1/\alpha \qquad (r = 1 \to n)$$

$$K'_{r,r} = \frac{K'_{r,r}}{\alpha C'_r} \qquad (r = 1 \to n)$$

$$K'_{r,r+1} = \frac{1}{\alpha\sqrt{C'_r \cdot C'_{r+1}}} \qquad (r = 1 \to n-1). \tag{19}$$

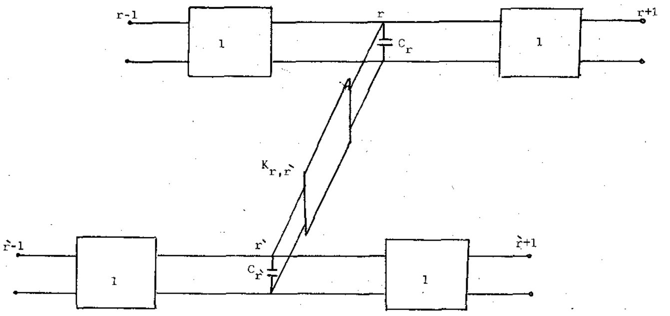


Fig. 8.

Due to the scaling process the input and the output ports are matched to $\alpha C_1'$ Ω rather than 1 Ω.

IV. Design Equations

By comparing the generalized comb-line section of Fig. 7 and the section of the low-pass prototype after the imposing of the frequency transformation given in (12), it can be shown that

$$Y_{r,r}=1 \qquad (r=1\to n)$$

$$Y_{r,r+1}=\frac{1}{\alpha\sqrt{C_r'\cdot C_{r+1}'}} \qquad (r=1\to n-1)$$

$$Y_{r,r'}=\frac{K_{r,r'}}{C_r'} \qquad (r=1\to n)$$

$$Y_r=1-\frac{1}{\alpha}\left\{\frac{1}{\sqrt{C_r'\cdot C_{r+1}'}}+\frac{1}{\sqrt{C_r'\cdot C_{r-1}'}}+\frac{K_{r,r'}}{C_r'}\right\} \qquad (r=2\to n-1)$$

$$Y_n=1-\frac{1}{\alpha}\left\{\frac{1}{\sqrt{C_{n-1}'C_n'}}+\frac{K_{n,n'}}{C_n'}\right\}$$

$$Y_1=1-\frac{1}{\alpha}\left\{\frac{1}{\sqrt{C_1'C_2'}}+\frac{K_{1,1'}}{C_1'}\right\}. \tag{20}$$

The input and the output sections can be designed to be matched to (αC_1) Ω by using capacitive coupling [10] as shown in Fig. 9

$$\frac{1}{\alpha C_1'}=j\omega C_p+\frac{j\omega C_s}{1+j\omega C_s} \tag{21}$$

$$C_s=\frac{1}{\omega\sqrt{\alpha C_1'-1}} \tag{22}$$

$$C_p=-\frac{\sqrt{C_1'\alpha-1}}{\omega\alpha C_{1'}} \tag{23}$$

and the center of passband

$$C_s=\frac{1}{\omega_0\sqrt{C_1'\alpha-1}} \tag{24}$$

$$C_p=-\frac{\sqrt{C_1'\alpha-1}}{\omega_0\alpha C_1'}. \tag{25}$$

C_p is negative and can be taken out of the first lumped capacitor.

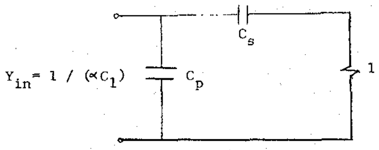


Fig. 9.

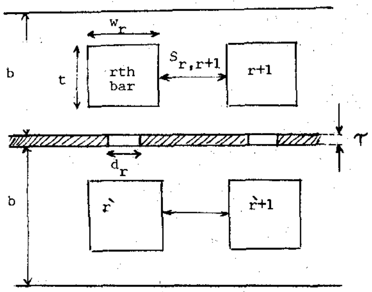


Fig. 10.

V. Design Example

This example intends to illustrate the idea of translating the element values of a given low-pass prototype network to physical dimensions.

Assume that the element values of the prototype network that meets the required specifications are

$$C_1'=0.8025 \qquad K_{1,1'}=0$$
$$C_2'=1.3878 \qquad K_{2,2'}=0.0596$$
$$C_3'=1.7467 \qquad K_{3,3'}=0.3438$$
$$C_4'=1.8939 \qquad K_{4,4'}=0.6825$$

i.e., the filter is of degree 8.

For the required comb-line filter we consider in this example: the center frequency $f_0=2$ GHz; the bandwidth $=2$ percent; length of each bar L_b at $f_0=\lambda/8$; the distance $b=2$ cm (see Fig. 10); the thickness of each bar $(t)=0.8$ cm; and the thickness of the separation plate $=\tau$.

Accordingly

$$L_b=1.873 \text{ cm}$$
$$\theta_0=0.25\pi \text{ rad}$$
$$\theta_1=0.2475\pi \text{ rad}$$
$$\theta_2=0.2525\pi \text{ rad}.$$

By using (12), the scaling factor is

$$\alpha=38.895.$$

The lumped capacitors C_r can be determined by using (18)

$$C_r=79.58 \text{ pF} \qquad (r=1,2,\cdots,8).$$

The shunt stubs admittances Y_r can be determined by using the corresponding expression in (20), $Y_1=0.976$, $Y_2=0.958$, $Y_3=0.964$, $Y_4=0.976$.

The main line coupling admittances $Y_{r,r+1}$ can be evaluated by using (20), $Y_{1,2}=0.0244$, $Y_{2,3}=0.0165$, $Y_{3,4}=0.0144$.

The cross-coupling admittances $Y_{r,r'}$ can be computed as indicated in (20), $Y_{1,1'}=0$, $Y_{2,2'}=0.0011$, $Y_{3,3'}=0.005$, $Y_{4,4'}=0.009$.

The admittances of shunt stubs and the main line couplings can be converted to physical lengths directly by using the graphs presented in [11]. By using these graphs it can be shown that $S_{1,2}=2$ cm, $S_{2,3}=2.1$ cm, $S_{3,4}=2.18$ cm, $W_1=1.00$, $W_2=0.975$ cm, $W_3=1.00$ cm, $W_4=1.04$ cm, where $S_{r,r+1}$ and W_r are shown in Fig. 10.

The dimensions of the slits can be evaluated by using the formula (see [12])

$$Y_{r,r}=(d_r^2/3840)(\pi/b)^2\left(\operatorname{sech}\left(\frac{\pi}{2}\cdot\frac{W_r}{b}\right)\right)^{-2}\exp\left(-\frac{\pi}{b}\cdot\frac{\tau}{d_r}\right)$$

where K_e is a complete elliptic integral [13]. The above equation can solved numerically to obtain d_r. The input and the output sections can be designed by using tapered conductors and the lumped capacitors can be realized by shaping the tip of each bar.

References

[1] J. D. Rhodes and I. H. Zabalawi, "Design of selective linear phase filters with equiripple amplitude characteristics," *IEEE Trans. Circuits Syst.*, vol. CAS-25 pp. 989–1000, Dec. 1978.

[2] R. J. Wenxel, "Synthesis of combine and capacitively loaded interdigital band-pass filters of arbitrary bandwidth," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 678–686, Aug. 1971.

[3] J. D. Rhodes, "A low-pass prototype network for microwave linear phase filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 290–301, June 1970.

[4] J. H. Cloete, "Tables for nonminimum-phase even-degree low pass prototype networks for the design of microwave linear-phase filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 123–128, Feb. 1979.

[5] J. D. Rhodes, "The generalized direct-coupled cavity linear phase filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 308–313, June 1970.

[6] J. D. Rhodes, "The generalized interdigital linear phase filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 301–307, June 1970.

[7] A. E. Williams and A. E. Atia, "Dual-mod canonical waveguide filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1021–1026, Dec. 1977.

[8] J. D. Rhodes, "The generalized interdigital networks," *IEEE Trans. Circuit Theory*, vol. CT-16, pp. 280–288, Aug. 1969.

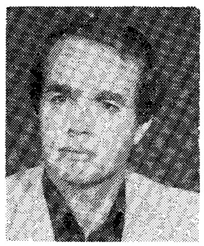
[9] J. D. Rhodes, "Lecture notes on microwave circuit theory," Univ. Leeds, 1976.

[10] S. B. Chon, "Direct-coupled resonator filters," *Proc. IRE* vol. 45, pp. 187–196, Feb. 1957.

[11] W. J. Getsinger, "Coupled rectangular bars between parallel plates," *IEEE Trans. Microwave Theory Tech.* vol. MTT-10, pp. 65–72, Jan. 1962.

[12] J. K. Shimizu and E. M. T. Jones, "Coupled-transmissionline directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 402–410, Oct. 1958.

[13] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions with Formulas, Graphs, Mathematical Tables.* New York: Dover, 1964.



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Combining the Powers from Multiple-Device Oscillators

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***Abstract*—This paper describes the results of power combining with multiple-device oscillators. A combiner circuit consisting of 3 oscillators and a directional coupler is analyzed. Conditions are set to obtain the maximum combining efficiency and a key approach is developed to control the frequency of the combiner. It is shown that the performance of the system is not seriously affected by the dissimilarity of the oscillators used in the combiner. A prototype 84-diode power combiner is constructed and total output power of 1.72 W with combining efficiency of 98.3 percent is obtained at 9.7 GHz. No fundamental limiting factor for the maximum number of devices to be combined was found.**

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I. Introduction

MICROWAVE POWER COMBINERS can be classified into two categories: 1) single circuit multiple-device structures, and 2) tree structures. In the former class, a number of devices contribute to the output power in a single circuit provided that the phase of the signal generated by each device is properly adjusted [1]–[3]. So

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